

F-term Braneworld inflation in Light of Five-year WMAP observations

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October 26, 2010

Abstract

We consider a supersymmetric hybrid inflation in the framework of the *Randall-Sundrum* type II Braneworld model. We derive an analytical expression for the scalar potential and find that the F-term dominates hybrid inflation process. We show that for some value of Brane tension we can eliminate the fine tuning problem related to coupling constant κ of the potential. We also calculate all known spectrum inflation parameters and show that observational bounds from WMAP5, BAO and SN observations are satisfied.

Keywords: *RS Braneworld, F-term inflation potential, Perturbation Spectrum, WMAP5.*

PACS numbers: 98.80. Cq

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1 Introduction

Recently there has been considerable interest on supersymmetric hybrid inflation[1] and its Braneworld extension in relation to observation[2]. Hybrid inflation was initially introduced to overcome the shortcomings of chaotic model[3], but cannot explain more complex questions of inflation theory, as reheating mechanism due to tachyon[4]. It turns out that one may consider hybrid inflation in supersymmetric theories. In fact, hybrid inflation looks more natural in supersymmetric theories rather than in non-supersymmetric ones[5]. Thus, it is very interesting to look for a generalized supersymmetric Braneworld inflation in relation with recent observations[6].

In Randall-Sundrum Braneworld scenario[7], our four-dimensional universe is considered living on a three-dimensional extended object (brane), embedded in a higher dimensional space (bulk). We shall study here some interesting cosmological implications of a supersymmetric hybrid brane inflation.

In this paper, we are interested on F-term effect on perturbation spectrum in Braneworld inflation. We have considered a Dvali superpotential which leads to the tree-level potential formed by an F-term and D-term[8]. Note that in most supersymmetric inflationary models only one of these terms dominates[9]. The case of F-term inflation, where F-term dominates, was considered for the first time in[10].

In the present work, we have shown that for some values of Brane tension we can eliminate the fine tuning problem related to coupling constant κ . We have also analyzed the perturbation spectrum for this potential, in particular the scalar spectral index, running of spectral index and ratio of scalar to tensorial amplitude perturbation was calculated. Our results are in good agreement with recent WMAP5 observations[6].

In the next section, we recall first, the foundation of a supersymmetric version of the hybrid inflation and different perturbation spectrum expressions in Randall-Sundrum type-II model(RS-II). In the section 3, we present our result for F-term inflation on the brane. A conclusion and perspective are given in the last section.

2 Supersymmetric Braneworld Inflation

2.1 Supersymmetric hybrid inflation

One of the most known one field inflationary models is the power law potential[3]

$$V(\sigma) = \frac{1}{2}m^2\sigma^2 + \frac{1}{4}\lambda\sigma^4 \quad (1)$$

where m is associated to scalar field σ . This potential is in a good agreement with observations, in particular for small λ [12].

$$\left(\frac{\delta T}{T_0}\right)_Q = 6,6 * 10^{-6} \Leftrightarrow \lambda = 6 * 10^{-14} \text{ et } m \lesssim 10^{13} GeV \quad (2)$$

This constraint on the constant λ , a dimensionless parameter, a priori in the order of unit, does not seem natural.

The hybrid inflation was introduced to take into account the anisotropy of temperature for λ in the order of unit. The model is based on the coupling of inflaton σ with a second scalar field χ as[13].

$$V(\sigma, \chi) = \frac{1}{4}\lambda(\chi^2 - M^2)^2 + \frac{1}{2}m^2\sigma^2 + \frac{1}{2}\lambda'\sigma^2\chi^2 \quad (3)$$

where M is associated to second scalar field χ . In this model, the inflaton σ is in slow rolling, while the field χ is responsible for the destabilization of potential which ends the inflation. The goal is to be able to take into account the smallness of the CMB anisotropies with potential coupling constants with more natural values ($\lambda, \lambda' \sim 0(1)$). Unfortunately, standard hybrid inflation leads to scalar index spectrum greater than 1[13].

In the supersymmetric hybrid Braneworld inflation, it was shown that the predictions of observable variables match with a recent results[14]. In this supersymmetric version, the potential has contributions from F-term and D-term[15]. Note that there exist two classes of hybrid inflation according to the origin of non-zero contribution of F-term or D-term to potential.

The F-term inflation is a generalization of the non supersymmetric version of hybrid inflation described by the potential eq.(3), with only two parameters. In this model, a superpotential consist of a coupling of two Higgs superfields $\Phi, \bar{\Phi}$ and a superfield S . It is given by[16]

$$W = \kappa S (-\mu^2 + \bar{\Phi}\Phi) \quad (4)$$

S is a scalar superfield singlet under a GUT's *gauge* group which is the inflaton field. κ and μ are two positive constants. The superpotential given by eq.(4) is the most general potential consistent with a continuous R-symmetry under which the fields transform as $S \rightarrow e^{i\gamma} S$, $W \rightarrow e^{i\gamma} W$ and $\bar{\Phi}\Phi$ is invariant.

In the following, we consider only the F-term contribution to inflation. Thus, the scalar potential is derived from the superpotential as

$$V = \kappa^2 |-\mu^2 + \bar{\phi}\phi|^2 + \kappa^2 |S|^2 (|\bar{\phi}|^2 + |\phi|^2) + \text{D-terms} \quad (5)$$

The ϕ and $\bar{\phi}$ are *Higgs* fields, which are a part of the superfields Φ and $\bar{\Phi}$ respectively. Note that these fields break $U(1)$ *gauge* symmetry in appropriate representations. Restricting ourselves to the D-flat direction and bringing $S, \bar{\phi}, \phi$ on the real axis: $S = \sigma/\sqrt{2}$, $\phi = \bar{\phi} = \chi/\sqrt{2}$ where σ, χ are normalized real scalar fields, we can obtain the potential which is similar to hybrid one[17]. Note that this potential does not contain the mass-term of scalar field σ which is of crucial importance to end inflation.

One way to generate the necessary slope along the inflationary trajectory is to include the one-loop radiative corrections on this trajectory ($\chi = 0$, $\sigma \succ \sigma_c = \sqrt{2}\mu$)[16]. In fact, SUSY breaking by the ‘vacuum’ energy density $\kappa^2\mu^4$ along this valley causes a mass splitting in the supermultiplets $\phi, \bar{\phi}$. We obtain a *Dirac* fermion with mass squared equal to $\frac{\kappa^2\sigma^2}{2}$ and two complex scalars with mass squared equal to $\frac{\kappa^2\sigma^2}{2} \pm \kappa^2\mu^2$. This leads to the existence of important one-loop radiative corrections to V on the inflationary valley which can be found from the *Coleman-Weinberg* formula[18]

$$\Delta V = \frac{1}{64\pi^2} \sum (-1)^{F_i} M_i^4 \ln \left(\frac{M_i^2}{\Lambda^2} \right) \quad (6)$$

where the sum extends over all helicity states i , F_i and M_i are the fermion number and mass of the i^{th} state respectively, and Λ is a renormalization mass scale. The one loop effective potential is then given by

$$V_{eff} = \kappa^2\mu^4 \left(1 + \frac{\kappa^2}{16\pi^2} \left(\ln \left(\frac{\kappa^2\sigma^2}{2\Lambda^2} \right) + \frac{3}{2} + \dots \right) \right) \quad (7)$$

Note in passing that the *Coleman-Weinberg* Potential was recently shown to be in good agreement with WMAP3 observations in the framework of non-supersymmetric GUTs[19]. Here we use this effective potential to study perturbation spectrum inflation in Braneworld scenario.

2.2 RS-II Braneworld model

The Randall-Sundrum Braneworld model was mainly studied and shown to give a very good agreement with recent observations[20, 21]. In this cosmological scenario, the metric projected onto the brane is a spatially flat *Friedmann-Robertson-Walker* with scale factor $a(t)$ and the Friedmann equation on the brane has the generalized form[22]

$$H^2 = \frac{8\pi}{3m_{pl}^2} \rho \left(1 + \frac{\rho}{2\lambda} \right) \quad (8)$$

where ρ is the energy density of the matter dominated in 3-brane, λ is the brane tension and m_{pl} is the *Planck* mass.

In Braneworld model, we consider that a scalar field is characterized by an energy density of the form $\rho = \frac{\dot{\phi}^2}{2} + V(\phi)$. The *Klein-Gordon* equation that describes the evolution of the scalar field is

$$\ddot{\sigma} + 3H\dot{\sigma} + V' = 0 \quad (9)$$

where $\dot{\sigma} = \frac{\partial \sigma}{\partial t}$, $\ddot{\sigma} = \frac{\partial^2 \sigma}{\partial t^2}$ and $V' = \frac{dV}{d\sigma}$.

We consider the slow-roll approximation ($\dot{\sigma}^2 \ll V(\sigma)$ and $\ddot{\sigma} \ll H\dot{\sigma}$) defined by parameters ϵ and η given by[23]

$$\epsilon = \frac{m_{pl}^2}{16\pi} \left(\frac{V'}{V} \right)^2 \frac{\left(1 + \frac{V}{\lambda} \right)}{\left(1 + \frac{V}{2\lambda} \right)^2} \quad (10)$$

$$\eta = \frac{m_{pl}^2}{8\pi} \left(\frac{V''}{V} \right) \frac{1}{\left(1 + \frac{V}{2\lambda} \right)} \quad (11)$$

where $V'' = \frac{d^2 V}{d\sigma^2}$.

During inflation $\epsilon \ll 1$ and $|\eta| \ll 1$. The end of inflation will take place for a field value σ_{end} such that $\max(\epsilon, |\eta|) = 1$.

The perturbation spectrum of inflation is characterized by[23]:

The scalar spectral index is presented by

$$n_s \simeq -6\epsilon + 2\eta + 1 \quad (12)$$

The power spectrum of the curvature perturbations given by

$$P_R(k) \simeq \frac{128\pi}{3m_p^6} \frac{V^3}{V'^2} \left(1 + \frac{V}{2\lambda} \right)^3 \quad (13)$$

The amplitude of tensor perturbations defined by[24]

$$P_g(k) \simeq \frac{128}{3m_p^4} V \left(1 + \frac{V}{2\lambda} \right) F^2(x) \quad (14)$$

where $x = Hm_p \sqrt{\frac{3}{4\pi\lambda}}$ and $F^2(x) = \left(\sqrt{1+x^2} - x^2 \ln \left(\frac{1}{x} + \sqrt{1+\frac{1}{x^2}} \right) \right)^{-1}$.

The ratio of tensor to scalar perturbations and the running of the scalar index presented respectively by

$$r(k) \simeq \left(\frac{m_p^2}{\pi} \frac{V'^2 F^2(x)}{V^2 \left(1 + \frac{V}{2\lambda} \right)^2} \right) \Big|_{k=k_*} \quad (15)$$

$$\frac{dn_s}{d \ln k} \simeq \frac{m_p^2}{4\pi} \frac{V'}{V} \frac{1}{\left(1 + \frac{V}{2\lambda}\right)} \left(3 \frac{\partial \varepsilon}{\partial \sigma} - \frac{\partial \eta}{\partial \sigma}\right) \quad (16)$$

Here, k_* is referred to $k = Ha$, the value when the Universe scale crosses the *Hubble* horizon during inflation.

Finally, the number of e-folds during inflation is[23]

$$N \simeq -\frac{8\pi}{m_{pl}^2} \int_{\sigma_*}^{\sigma_{end}} \frac{V}{V'} \left(1 + \frac{V}{2\lambda}\right) d\sigma \quad (17)$$

where the subscripts $*$ and end are used to denote the epoch when the cosmological scales exit the horizon and the end of inflation, respectively.

3 F-term inflation in the light of WMAP5

In this section, we introduce the F-term hybrid inflation by using Braneworld model. Note that the F-term inflation was introduced to solve the blue spectrum problem ($n_s > 1$)[13] and was shown to give a good values of perturbation spectrum[14]. In the present work we give an exact evaluation of various inflation parameters.

The inflation end at $\sigma = \sigma_c$ if $\sigma_c \geq \sigma_{end}$. These parameters are given by

$$|\eta| = 1 \quad \Rightarrow \quad \sigma_{end} = \frac{m_{pl} \kappa^2 \mu^2}{8 (\pi)^{3/2}} \sqrt{\frac{1}{\kappa^2 \mu^4 \left(\frac{k^2 \mu^4}{2\lambda} + 1\right)}} \quad (18)$$

$$\frac{\partial^2 V}{\partial \chi^2} \Big|_{\chi=0} = 0 \quad \Rightarrow \quad \sigma_c = \sqrt{2} \mu \quad (19)$$

this is equivalent to the condition

$$\sigma_c \geq \sigma_{end} \Rightarrow \lambda \leq \frac{\kappa^2 \mu^4}{\frac{m_{pl}^2 \kappa^2}{64 \mu^2 \pi^3} - 2} \quad (20)$$

thus, we have obtained an upper limit for the brane tension λ for some value of potential parameters κ and μ .

From eq.(17), we evaluate the corresponding inflaton field value σ_* as

$$\sigma_*^2 \simeq \frac{N \kappa^2 m_{pl}^2}{32 \pi^3 \left(1 + \frac{\kappa^2 \mu^4}{2\lambda}\right)} + 2 \mu^2 \quad (21)$$

On the other hand, the combination of WMAP5, BAO, and SN data gives the following results[6]

$$0.9392 < n_s < 0.9986 \quad (22)$$

$$r < 0.20 \quad (23)$$

$$-0.0728 < \frac{dn_s}{d \ln k} < 0.0087 \quad (24)$$

$$P_R(k) = 2.457 \times 10^{-9} \quad (25)$$

From eq.(20) where $\kappa > \sqrt{128 \pi^3} \frac{\mu}{m_{pl}}$, and taking into account of $\mu = 4 \times 10^{-4} m_p$ [14] and $m_{pl} = 1.2 \times 10^{19} GeV$, we derive a numerical limit of a potential coupling constant $\kappa \gtrsim 0.02$. This result allows us to eliminate the fine tuning problem.

For $\kappa = 0.03$, and $N = 55$ for example we obtain from eq.(20) an upper bound for Brane tension as

$$\lambda \leq 5.337 \times 10^{-18} m_{pl}^4 \quad (26)$$

Using $\lambda = 2.985 \times 10^{-18} m_{pl}^4$ which verifies the inequation (26), we can obtain

$$P_R(k) \simeq 2.4 \times 10^{-9} \quad (27)$$

which is practically the same value provided by the observation.

Using the obtained value of Brane tension λ , we can estimate the fundamental *Planck* scale M_5

$$M_5 \sim 10^{-3} m_{pl} \quad (28)$$

The fundamental *Planck* scale may therefore be significantly below the effective *Planck* scale. This could provide partially a solution to the hierarchy problem[25].

We note that a large interval of variation of N can reproduce the observation results as shown in the following table

<i>Inflation parameters</i>	n_s	r	$\frac{dn_s}{d \ln k}$
$N = 50$	0.9610	$0.43 * 10^{-5}$	-0.0015
$N = 55$	0.9645	$0.39 * 10^{-5}$	-0.0012
$N = 60$	0.9674	$0.36 * 10^{-5}$	-0.001

We can also remark that the scalar spectral index n_s and the running of the scalar spectral index $\frac{dn_s}{d \ln k}$ increase according to N , whereas, the ratio of tensor to scalar perturbations r decreases.

4 Conclusion

In this paper, we have studied F-term hybrid inflation in framework of the Randall-Sundrum Braneworld typeII model. We have derived the effective potential by introducing the *Coleman-Weinberg* correction to evaluate various parameters spectrum perturbation. We have shown that the fine-tuning problem (very small value of coupling constant \varkappa) is solved and an upper limit for the brane tension λ was obtained. Various spectrum perturbation parameters were also calculated with a particular choice of \varkappa and λ . In particular, our results for power spectrum of the curvature perturbations $P_R(k)$ coincide with recent observations. We have also shown a good compatibility of others inflation perturbation parameters namely the scalar spectral index n_s , the ratio of tensor to scalar perturbations r and the running of the scalar index $\frac{dn_s}{d \ln k}$ with recent observations.

References

- [1] Richard A. Battye, Bjorn Garbrecht, Adam Moss "Constraints on Supersymmetric Models of Hybrid Inflation" *JCAP 0609 (2006) 007*
- [2] Mansoor ur Rehman, V. N. Senoguz, Qaisar Shafi "Supersymmetric And Smooth Hybrid Inflation In The Light of WMAP3" *Phys.Rev.D75(2007)043522*.
- [3] A. D. Linde, Particle Physics and Inflationary Cosmology (Harwood, Chur, Switzerland, 1990).
- [4] Yun-Song Piao, Qing-Guo Huang, X. Zhang, Y.Z. Zhang, "Nonminimally coupled tachyon and inflation," *Phys.Lett. B570 (2003) 1-4*.

- [5] D.H.Lyth and A.Riotto, "Comments on D-term inflation," *Phys. Lett. B* 412 (1997) 28.
- [6] E. Komatsu et al. "Five-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Interpretation," *Astrophys.J.Suppl.*180:(2009)330-376.
- [7] L.Randall and R.Sundrum, "An alternative to compactification," *Phys. Rev. Lett.* 83 (1999) 4690.
- [8] Tomohiro Matsuda "F-term, D-term and hybrid brane inflation" *JCAP* 0311 (2003) 003.
- [9] R.Jocher and M.Sakellariadou,"Constraint on Supersymmetric Grand Unified Theories from Cosmology," *JCAP* 0503 (2005) 004.
- [10] P.Binetruy and G.R.Dvali, "D-term inflation," *Phys. Lett. B* 388 (1996) 241.
- [11] G.R.Dvali, Q.Shafi and R.K.Schaefer, "Large scale structure and supersymmetric inflation without fine tuning" *Phys. Rev. Lett.* 73 (1994) 1886.
- [12] S. M. Leach and A. R. Liddle, *Phys. Rev. D* 68 123508(2003).
- [13] A. D. Linde, *Phys. Rev. D*49, 748 (1994).
- [14] H.Boutaleb-J, A.Chafik and A.L.Marrakchi, "Supersymmetric hybrid inflation in the braneworld scenario" *Phys. Lett. B* 574 (2003) 89.
- [15] Tomohiro Matsuda "F-term, D-term and hybrid brane inflation" *JCAP* 0311 (2003) 003
- [16] G. Dvali, Q. Shafi, and R. Shaefer, *Phys. Rev. Lett.* 73, 1886 (1994) .
- [17] A. D. Linde, *Phys. Lett. B* 259, 38 (1991)
- [18] Coleman, S. and Weinberg, E. *Phys. Rev. D* 7 (1973) 1888.
- [19] Q. Shafi, V. N. Senoguz, "Coleman-Weinberg Potential In Good Agreement With WMAP " *Phys.Rev.D*73:127301,2006
- [20] M. Bennai, H. Chakir, Z. Sakhi, "Aspects of inflation in Brances physics" *Afr.J.Math.Phys.*1 (2004) 185-190
- [21] R. Zarrouki, Z. Sakhi and M. Bennai, "WMAP5 Observationnal Constraints on Braneworld New Inflation Model," *Int.J.Mod.Phys.A*25 (2010) 171-183.
- [22] P. Binetruy, C. Deffayet, U. Ellwanger, and D. Langlois, *Phys. Lett. B*477, 285 (2000).
- [23] R.Maartens, D.Wands, B.Basset, and I.Heard, "Chaotic inflation on the brane," *Phys.Rev.D* 62 (2000) 041301.
- [24] S. Tsujikawa, D. Parkinson and B. A. Bassett, *Phys. Rev. D* 67, 083516 (2003).
- [25] N.Arkani-Hamed, S.Dimopoulos and G.R.Dvali, "The hierarchy problem and new dimensions at millimeter," *Phys. Lett. B* 429 (1998) 263.